Unit 3: Triangles

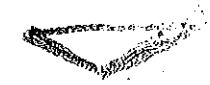
4-1 Classifying Triangles

Math Vocabulary:

1. Triangle: 3 sided polygon with 3 angles
2. Segment: sides of a triangle
3. Vertices: point where the segments intersect

Classifying Triangles by Angles:

What are the 4 types? Describe them:

- Acute: 3 angles less than 90° 
- Obtuse: 1 angle greater than 90° 
- Right: 1 angle is 90° 
- Equiangular: 3 equal angles, 60° each

Classifying Triangles by Congruent Sides:

What are the 3 types? Describe them:

- Isosceles: 2 sides are congruent 
- Equilateral: 3 sides are congruent 
- Scalene: 3 unequal sides

Congruent △s: Corresponding angles and sides are congruent

Right Triangles:

Name of the longest side:

Pythagorean Theorem:

Naming Triangles:

Triangles can be classified by their angle measures or the measures of their sides.

Which combinations of labels are possible?

sides: AB, AC, BC

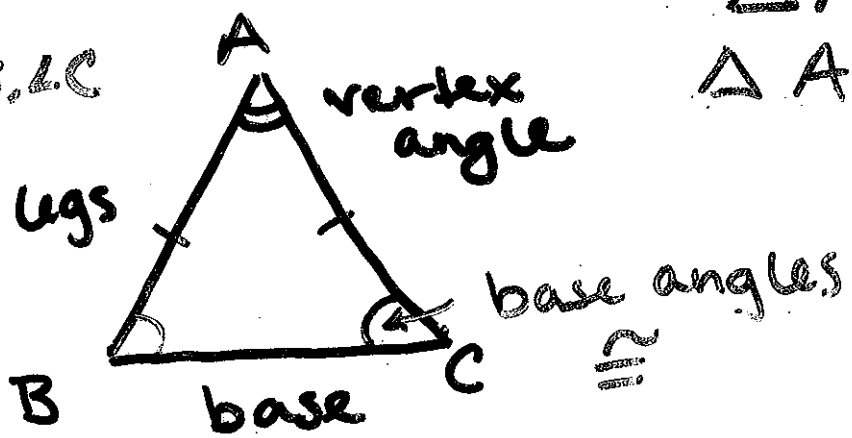
▲ Isosceles Δs

▲ Angles:

▲ ∠A, ∠B, ∠C

△ABC

△ACB



▲

▲

▲

Impossible?

▲

▲

$AC = x + 5$ $AB = 2x + 1$

Find length of AC

$AC \cong AB$ $x + 5 = 2x + 1$

$x = 4$

$AC = x + 5 = 4 + 5 = 9$

Review:

1. If a triangle has three unequal sides it is a _____ triangle.
2. A triangle which has three equal angles is a _____ triangle.
3. A triangle with (at least) two equal sides is a _____ triangle.
4. Two triangles that are equal are called _____ triangles.
5. A triangle with all three sides of equal length is a _____ triangle.
6. A triangle with an angle of 90° is a _____ triangle.
7. A triangle in which all three angles are less than 90° is a _____ triangle.
8. A triangle in which one of the angles is greater than 90° is a _____ triangle.
9. What is the Hypotenuse of a right triangle?
10. The Pythagorean Theorem states ...

4-2 Measuring Angles

Theorems and Corollaries:

Theorem 4-1 Angle Sum Theorem: The sum of the measure of the angles of a triangle is 180°

Theorem 4-2 Third Angle Theorem: If 2 angles of a △ are ≅ to 2 ∠s of a 2nd triangle, then the 3rd ∠ of each △ is ≅

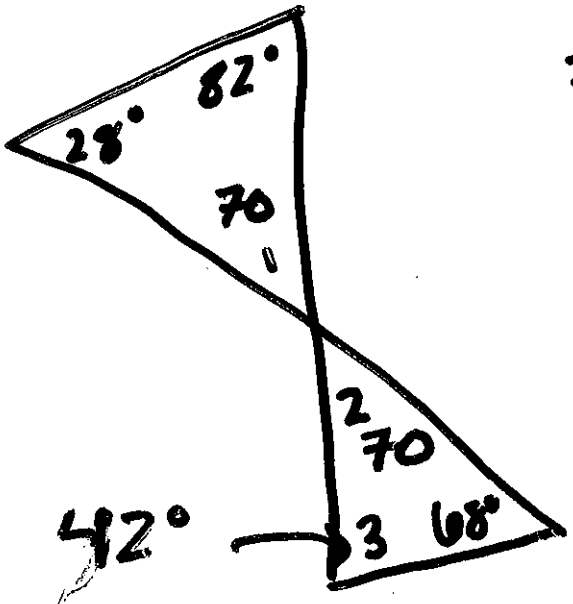
Theorem 4-3 Exterior Angle Theorem: The measure of

Corollary 4-1: An exterior ∠ of a △ is = to the sum of the measures of the 2 remote interior angles

Corollary 4-2:

Corollary 4-1: The acute angles of a right △ are complementary

Corollary 4-2: There can be at most 1 right or obtuse angle in a triangle.



Find m∠1
m∠2
m∠3 32°

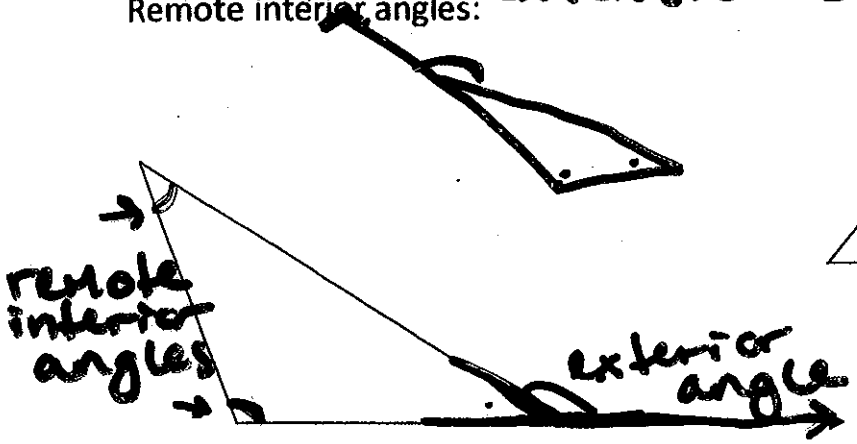
70+68+m∠3 = 180

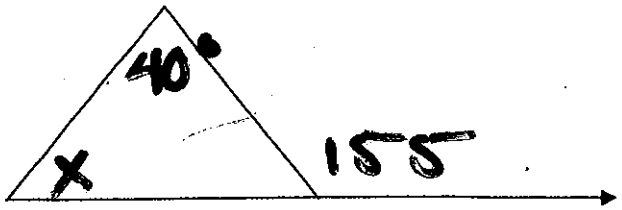
28+82+m∠1 = 180
110 + m∠1 = 180
m∠1 = 70

Example of an Exterior Angle:

Exterior angle: Formed by 1 side of a △ and an extension of another side

Remote interior angles:



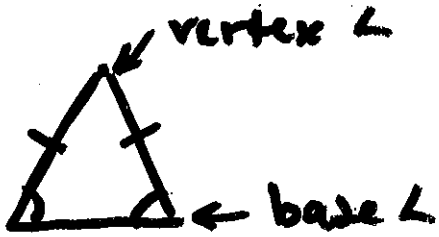


$x + 40 = 155$

$x = 115°$

4-6 Isosceles Triangles

Isosceles Triangles



Theorem 4-6 Isosceles Triangle Theorem:

If 2 sides of a △ are ≅, then the ∠s opposite those sides are ≅

Theorem 4-7:

If 2 ∠s of a △ are ≅, then the sides opposite these ∠s are ≅

Equilateral and Equiangular Triangles

Corollary 4-3: A △ is equilateral if and only if it is equiangular

Corollary 4-4: Each ∠ of an equilateral △ is 60°

Activity:

Use a compass and a straightedge to construct an equilateral triangle, an isosceles triangle, and a scalene triangle. Cut out each triangle.

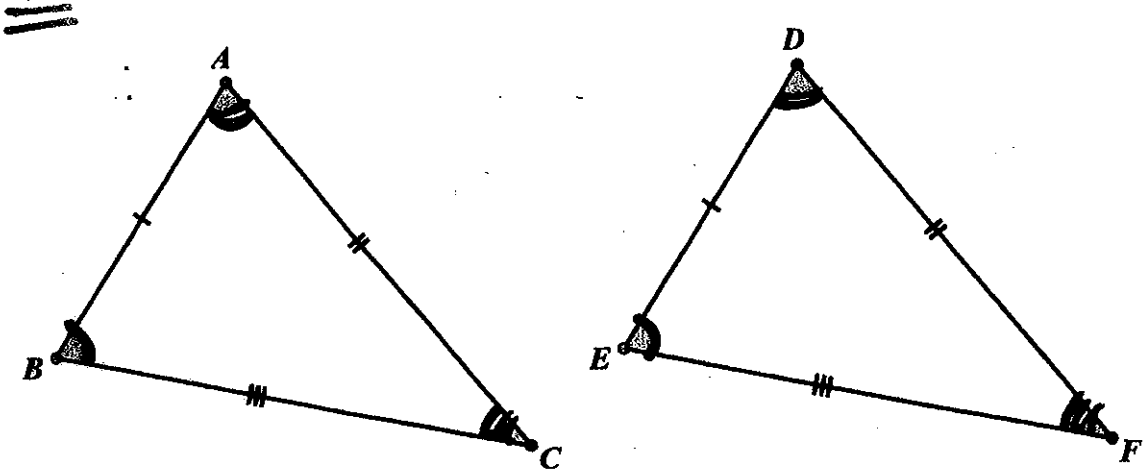
1. How many ways can an equilateral triangle be folded to form two congruent triangles?
2. How many ways can an isosceles triangle be folded to form two congruent triangles?
3. How many ways can a scalene triangle be folded to form two congruent triangles? Corresponding Parts Congruent Triangle are Congruent

4-3, 4-4 & 4-5 Congruent Triangles

Congruent Triangles (CPCTC): Two △s are ≅ iff (if and only if) their Corresponding parts are congruent

Each triangle has 6 parts: 3 vertices and 3 sides. All 6 parts must be congruent.

When you write congruence statements, you **MUST** list the vertices in the correct order.



Name the congruent parts:

$\angle A \cong \angle D$ $\quad \overline{AB} \cong \overline{DE}$

$\angle B \cong \angle E$ $\quad \overline{AC} \cong \overline{DF}$

$\angle C \cong \angle F$ $\quad \overline{BC} \cong \overline{EF}$

Write the congruence statement:

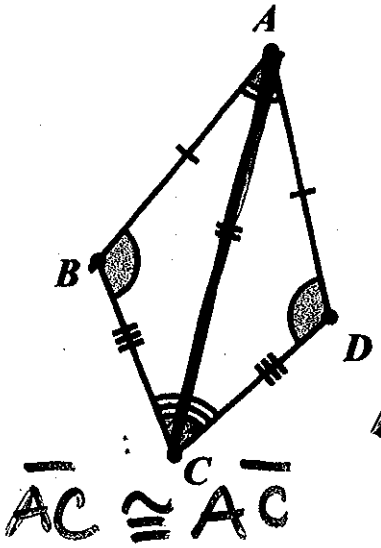
$\triangle ABC \cong \triangle DEF$ $\quad \triangle CAB \cong \triangle FDE$

NOT $\triangle ABC \cong \triangle FDE$

Congruence Transformations:

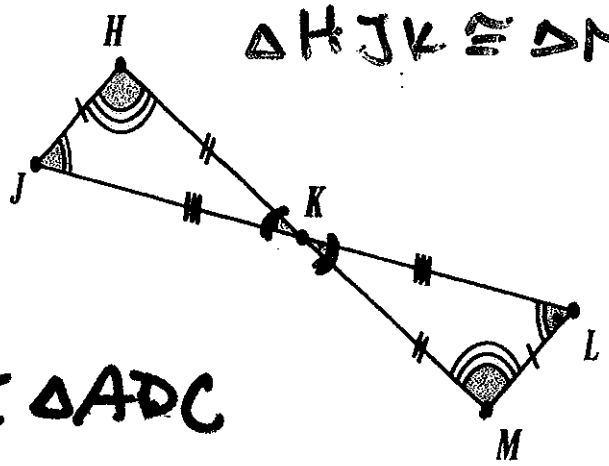
Theorem 4-4: Congruence of triangles is reflexive, symmetric, and transitive

$\triangle DEF \cong \triangle HJK$



$\overline{AC} \cong \overline{AC}$

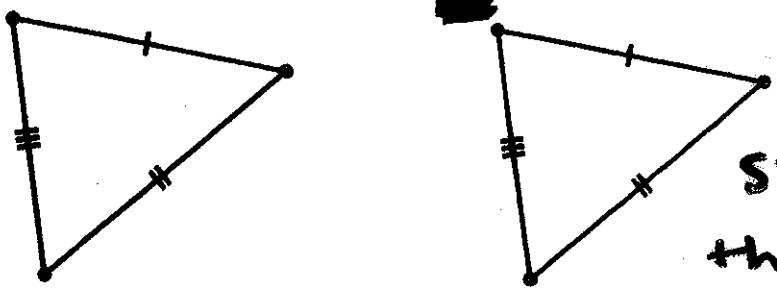
$\triangle ABC \cong \triangle ADC$



$\triangle HJK \cong \triangle MLK$

Theorems and Postulates: ways to show congruence

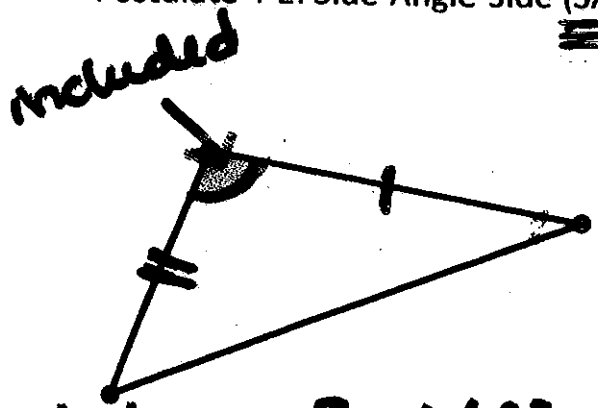
Postulate 4-1: Side-Side-Side (SSS Postulate)

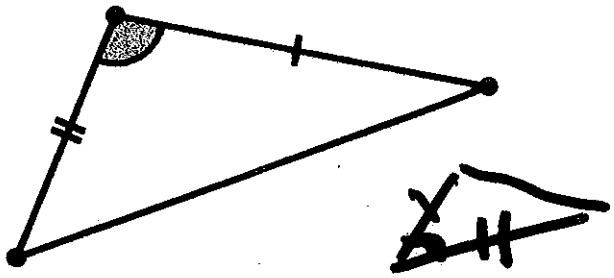


If the sides of $\triangle$ are $\cong$ to the sides of the 2nd $\triangle$, then the $\triangle$s $\cong$

Label and write the congruence statement:

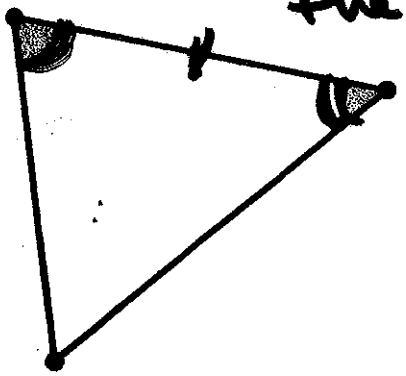
Postulate 4-2: Side-Angle-Side (SAS Postulate) If 2 sides and the included angle are ≅

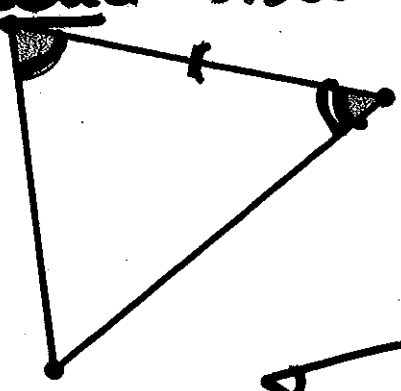


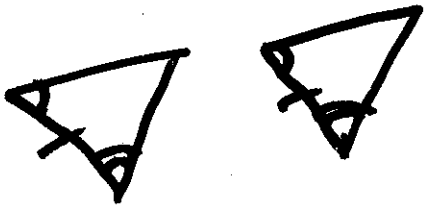


Label and write the congruence statement:

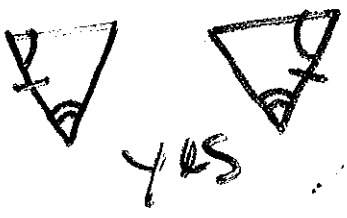
Postulate 4-3: Angle-Side-Angle (ASA Postulate) If 2 angles and the included side ≅





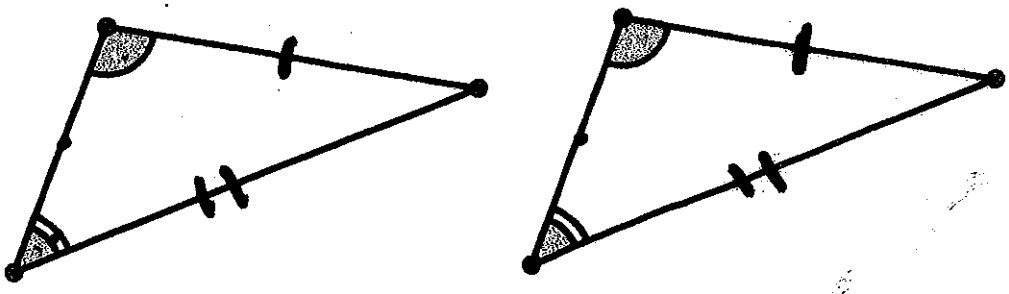


Label and write the congruence statement:

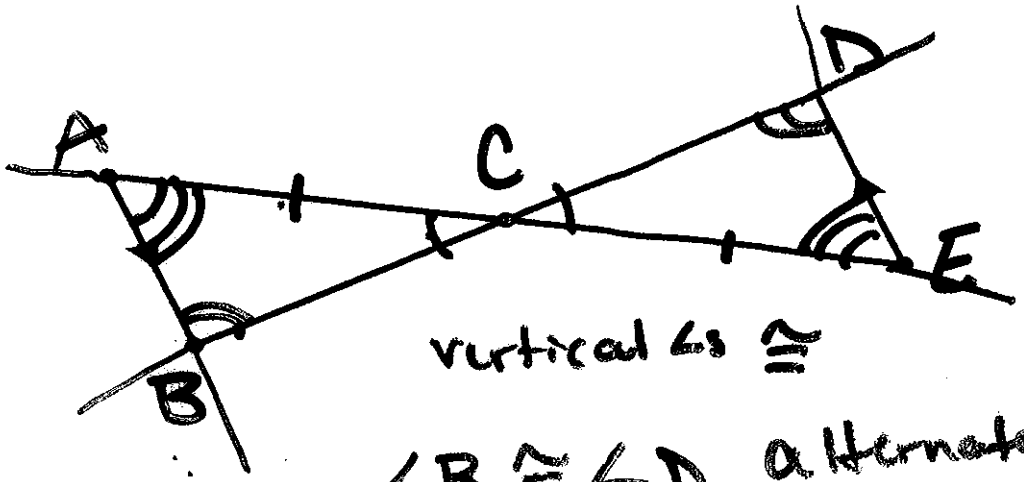


SSS
SAS
ASA

Theorem 4-5: Angle-Angle-Side (AAS Theorem) two angles and a non-included side of Δs are ≅



Label and write the congruence statement:



$AC \cong CB$

vertical ∠s ≅

$\angle B \cong \angle D$ alternate interior
$\angle A \cong \angle E$ "

AAS or ASA

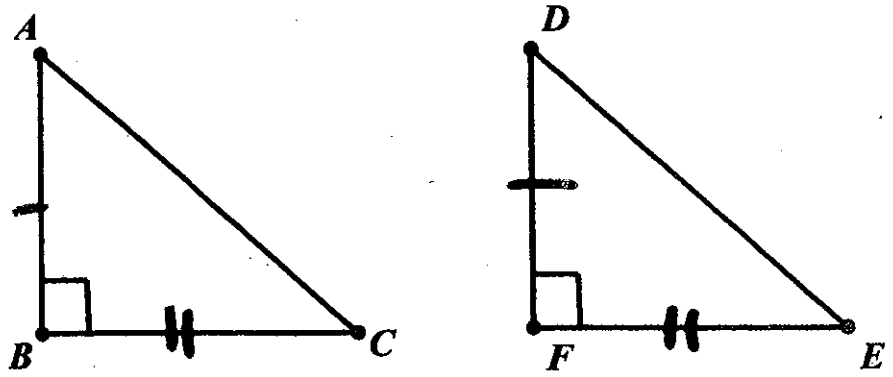
never SSS → similarity NOT ≅
never AAA →

$\triangle CAB \cong \triangle CED$

5-2 Right Triangles

Theorem 5-5: Leg-Leg (LL Theorem)

If the legs of one right triangle are congruent to the corresponding legs of another right triangle, then the triangles are congruent.

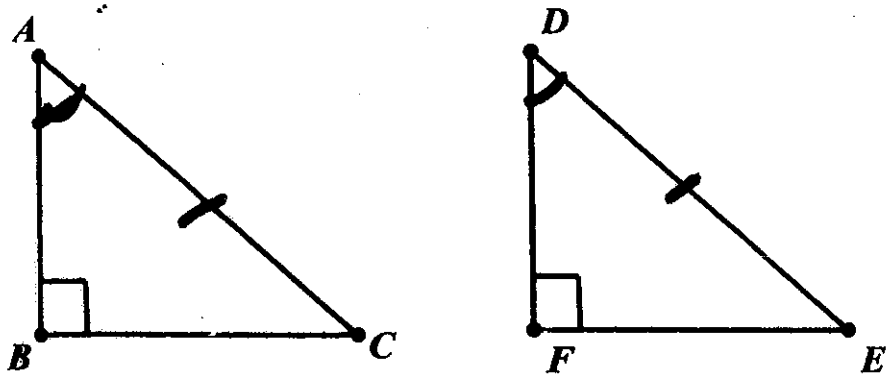


called LL "leg-leg"

What other theorem is this like? SAS

Theorem 5-6: Hypotenuse-Angle (HA Theorem)

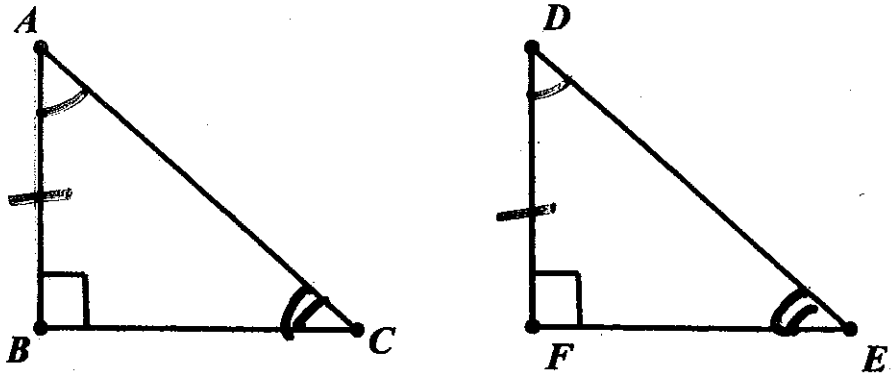
If the hypotenuse and an acute angle of one right triangle are congruent to the hypotenuse and an acute angle of another right triangle, then the triangles are congruent.



What other theorem is this like? AAS

Theorem 5-7: Leg-Angle (LA Theorem)

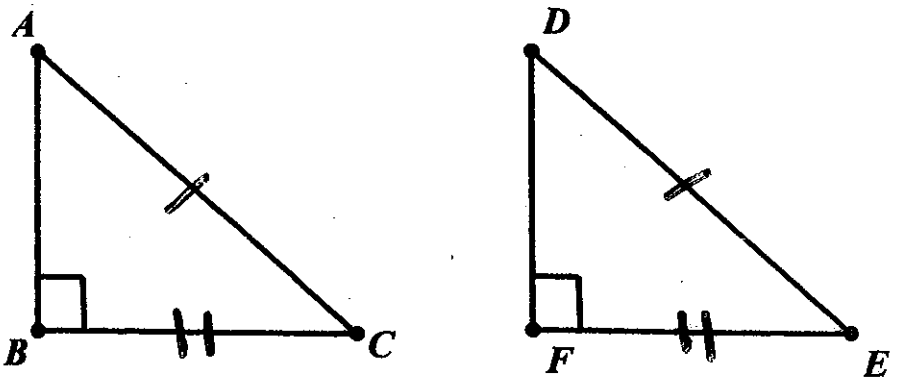
If one leg and an acute angle of one right triangle are congruent to the corresponding leg and acute angle of another right triangle, then the triangles are congruent.



What other theorem is this like? ASA, AAS

Postulate 5-1: Hypotenuse-Leg (HL Theorem)

If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and corresponding leg of another right triangle, then the triangles are congruent.



Rt. Δ use H.L (not ASS)